MODE COMPUTATION IN LONG TAPERED MULTI-CELL LINEAR ACCELERATOR STRUCTURES USING THE GSM METHOD

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Abstract

For a proper design of linear colliders it is important to know the resonant modes corresponding to higher order dipole passbands of long tapered multi-cell structures. Grid-oriented codes, as e.g. the MAFIA program package, cannot be used for the analysis of such structures. In this contribution, the accurate and numerical efficient generalized scattering matrix method is applied to the computation of these modes. The sixth dipole passband of the 180-cell accelerating structure used for the S-band linear collider at DESY is extensively being investigated with the proposed method. The calculations predict that this passband is especially dangerous for a stable operation of the collider which will lead to a change of the current structure design. The validity of the developed code is confirmed by comparing the results which have been obtained from the MAFIA program package for a 36-cell structure with those of our method. Furthermore a special numerical technique is suggested allowing a reliable computation of the so-called trapped modes which often occur in tapered multi-cell structures.

Introduction

At several high energy physics laboratories around the world strong efforts are currently made to design an e^+e^- linear collider with an initial center of mass energy of about 500 GeV [1]. In most of the designs trains of bunches are accelerated by long tapered multi-cell structures which are similar to disc-loaded circular waveguides. In order to avoid deflections of the beam which decrease the efficiency of the collider or even may lead to cumulative beam instabilities, one has to control the wakefield excited by previous bunches in the train. Higher order modes corresponding to the first and sixth dipole passband are mainly responsible for this phenomenon.

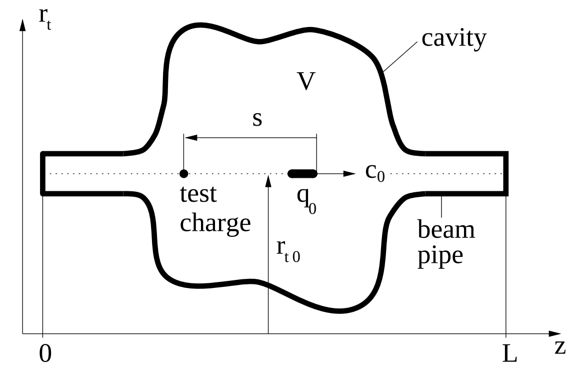

Figure 1: Bunch which passes through a cavity followed by a test charge.

The effect of these modes can be investigated by the so-called loss parameter [2]. In order to illustrate the meaning of this quantity let us consider Fig. 1. It shows a bunch of particles with the total charge q_0 passing through a cavity parallel to the z-axis at the speed of light c_0. The cavity with volume V extends from $z = 0$ to $z = L$; and the transverse bunch coordinate is $\boldsymbol{r}_{t0}$. Now assume that this bunch is followed by a small test charge on the same path in a distance s. The longitudinal wake potential then is defined as:

$$W_{\|}(\boldsymbol{r}_{t0}, s) = \frac{1}{q_0} \int_{z=0}^{L} E_z\left(\boldsymbol{r}_{t0}, z, t = \frac{s+z}{c_0}\right) dz \quad (1)$$

$W_{\|}$ describes the change of the longitudinal component of the momentum of the test charge due to the wakefield driven by the leading bunch. E.g., the MAFIA program package [3] which is based on a grid discretization of Maxwell's equations can be used to compute the wake potential of arbitrarily shaped cavities. Nevertheless, it turns out that the application of grid-oriented codes to real accelerating structures which are weakly tapered is numerically inefficient because a tremendous number of mesh points is required for an accurate model of the boundary of such structures.

In [2], it has been demonstrated that the wake potential $W_{\|}^{\delta}$ of a point charge traversing a closed cavity is just given by an infinite sum over the resonant modes of the cavity

$$W_{\|}^{\delta}(\boldsymbol{r}_{t0}, s) = 2\sum_{n}^{\infty} k_n(\boldsymbol{r}_{t0}) \cos\left(\frac{\omega_n s}{c_0}\right) \;,\; s > 0 \quad (2)$$

where k_n and ω_n denote the loss parameter and the resonant frequency of the nth cavity eigenmode, respectively. The loss parameter reads

$$k_n(\boldsymbol{r}_{t0}) = \frac{|V_n(\boldsymbol{r}_{t0})|^2}{4U_n} \quad , \quad (3)$$

$$V_n(\boldsymbol{r}_{t0}) = \int_{z=0}^{L} E_{zn}(\boldsymbol{r}_{t0}, z) \exp\left(j\frac{\omega_n z}{c_0}\right) dz \quad , \quad (4)$$

$$U_n = \frac{\varepsilon_0}{2} \int_V |\boldsymbol{E}_n|^2 \, dV \quad , \quad (5)$$

where V_n and U_n denote the "complex voltage seen by the test charge" and the total energy, respectively, corresponding to the nth cavity eigenmode. Note that even for $s < L$, which means that the leading bunch and the test charge are simultaneously inside the cavity for a certain period of time, the irrotational cavity eigenfunctions [4] do not appear in eq. (2) [5]. The wake potential of a closed accelerating structure can consequently be calculated from its resonant modes only. In reality the beam pipe is open. Nevertheless, formula (2) can

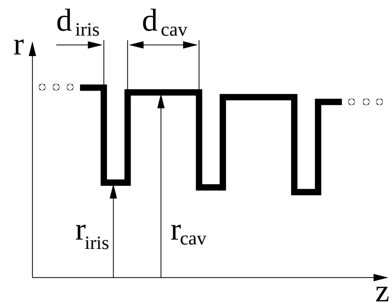

Figure 2: Longitudinal section of a tapered multi-cell linear accelerator structure.

d_{iris}	d_{cav}	First cell		Last cell	
		r_{iris}	r_{cav}	r_{iris}	r_{cav}
5.00	28.33	16.11	40.65	11.65	39.26

Table 1: Geometry of the 180-cell S-band structure. All dimensions are given in mm and are linearly tapered between the first and the last cell.

also be used to describe the spectral components of the wake potential below the cut off frequency of the beam pipe. Therefore the beam pipe has to be modeled sufficiently long so that the cavity modes do not depend on the position of the short.

The generalized scattering matrix (GSM) method has been proved accurate and numerically efficient for the investigation of a large variety of waveguide and cavity problems [6]. In the field of linear accelerators, this method has also successfully been applied. E.g., the beam loading in a tapered X-band accelerating structure driven by monopole modes has been analyzed using the GSM method for the computation of the corresponding resonant modes [7].

Dipole modes corresponding to the first passband of a 180-cell structure which has been designed for the S-band linear collider at DESY have been studied in detail in [8]. The geometry of this structure is given in Fig. 2 and Table 1. However, in [8] only the first dipole passband has been considered. Furthermore, instead of the original structure a simplified model has been assumed which consists of 30 packages containing 6 identi-

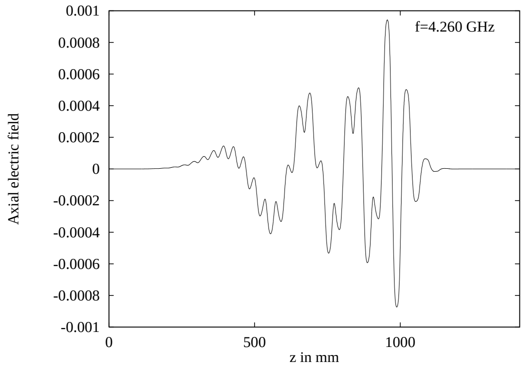

Figure 3: Axial electric field of a trapped mode corresponding to the first dipole passband of the 36-cell structure.

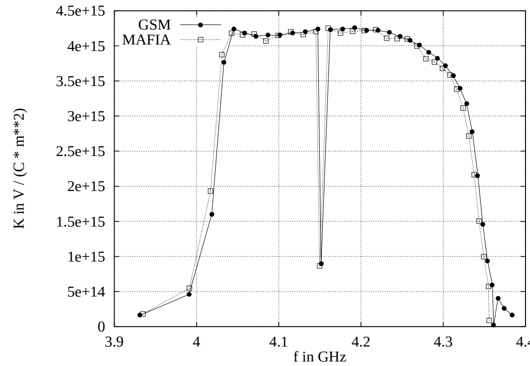

Figure 4: Comparison of the loss parameters corresponding to the GSM method and MAFIA for the first dipole passband of the 36-cell structure.

cal cells each. The numerical results have demonstrated that this assumption leads to a wide scattering of the loss parameters which is however an artifact of the calculation model.

Basic Formulation

In this contribution the sixth passband of the 180-cell structure is studied using a GSM code which has originally been developed for the analysis of gyrotron cavities [9]. Recently, the computation of loss parameters has also been included in this code. A HP735 workstation has proved to be sufficient for the numerical calculations. Therefore our code seems to be much more numerically efficient than that presented in [8] which requires a supercomputer even for the analysis of the simplified model of the 180-cell structure.

An unusual feature of tapered multi-cell structures are the so-called "trapped modes". In order to illustrate this phenomenon, Fig. 3 presents the electric field of such a mode corresponding to the first dipole passband of a 36-cell structure which has also been analyzed in [10]. Due to the dispersion characteristics of the different cells, most of the resonant modes have no contact with the end cells, see Fig. 3. This makes the numerical evaluation somewhat tedious: if we apply the resonance condition for the search of the cavity modes at a certain cross section of the structure, only those modes which contribute significantly to the field there are found. Therefore we have to look for cavity modes at various cross sections. Pursuing this procedure, it cannot be avoided that most of the modes are multiply computed. For the 180-cell structure it has been found that it is sufficient to search for cavity modes at six cross sections which are equally spaced along the structure.

Numerical Results

The validity of the proposed method has been checked by comparing the results which have been obtained from the MAFIA program package for the 36-cell structure [10] with those of our method. From Fig. 4, it can be concluded that the agreement between the results corresponding to both methods is excellent.

For a qualitative estimate of the loss parameters covering several higher order dipole passbands we have analyzed a 30-cell structure which consists of every sixth cell of the original structure. The results presented in Fig. 5 underline the importance of the sixth dipole passband which contains the modes with the highest loss parameters.

Fig. 6 shows the results of a detailed investigation of the sixth dipole passband of the original structure in the immediate vicinity of the mode with the highest loss parameter. An extensive study of convergence has been carried out in order to demonstrate the accuracy of the method. It has turned out that the loss parameters are stable if

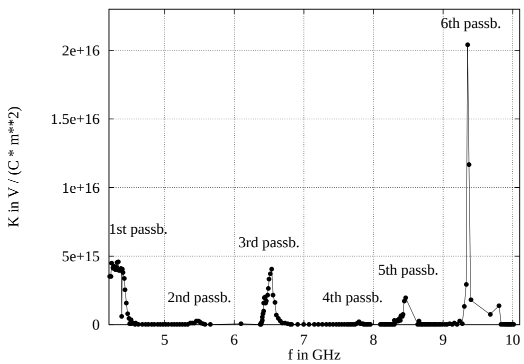

Figure 5: Loss parameters corresponding to dipole modes of the 30-cell structure in the frequency range from 4.2 GHz to 10.2 GHz.

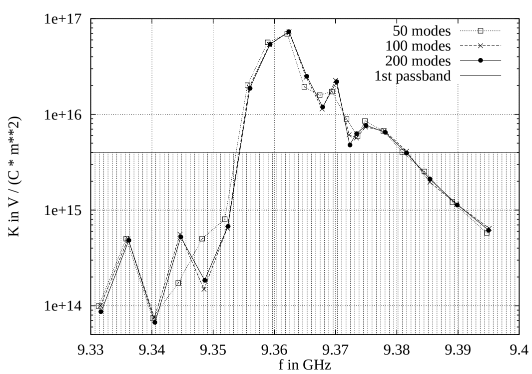

Figure 6: Loss parameters corresponding to the sixth passband of the 180-cell structure.

more than 100 waveguide modes are used in the GSM method.

The level of the loss parameters corresponding to the first dipole passband ($\approx 4 \cdot 10^{15}$ V/(C m^2)), which is also given in Fig. 6, is about 20 times less than the peak loss parameter observed in the sixth dipole passband ($\approx 8 \cdot 10^{16}$ V/(C m^2)), which has serious consequences for beam dynamics and the future structure design. In order to suppress these modes an additional tapering of the iris thickness is currently being investigated.

Conclusions

The GSM method has been applied to the computation of various dipole passbands corresponding to multi-cell linear accelerator structures. A special numerical technique has been developed for the determination of trapped modes. A detailed study of the sixth dipole passband of the 180-cell structure used for the S-band linear collider at DESY has shown, that the maximum loss parameter corresponding to this passband is about 20 times as high as that of the first passband.

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